



Making Sense: Teaching and Learning Mathematics with Understanding

Feb. 28th, 2005

"Almost all, who have ever fully understood arithmetic, have been obliged to learn it over again in their own way."

Warren Colburn



Making Sense

- Our purpose is to provide a framework within which teachers can reflect on their own practice, and think again about what it means to teach for understanding.
- Classrooms that facilitate mathematical understanding share some core features.
 - Classrooms that promote understanding can look very different on the surface and still share some common core features.
 - Designing classrooms for understanding does not mean conforming to a single, highly prescribed method of teaching



Agenda/ Overview

Feb. 28th- Jon Manon

- What is mathematical knowledge? (Piaget)
 - Conventional knowledge
 - Mathematical knowledge means making sense
 - Mathematical logic requires students to make logical connections
- What is strategic competence?
- What kinds of classrooms encourage sense making?
- What should our classrooms look like?
- How important are the tasks we select?
- What does deproblematizing math mean?
- How can we create communities of learners that solve mathematical problems?



Understanding

- Mathematics should be based on the understanding of fundamentals in a structured pattern.
- Two kinds of understanding
 - Relational understanding as knowing what to do and why and
 - Instrumental understanding as knowing what to do or the possession of a rule and ability to use it.
 - Understanding is something that is always changing and ever growing.



Deep of understanding vs. regurgitation of facts

- *Mathematics reveal hidden patterns that help us understand the world around us
Mathematics is the science of pattern and order... Its domain is Numbers, chance, form, algorithms, and change...
Mathematics relies on logic rather than on observation as its standard of truth, yet employs observation, simulation, and even experimentation as means of observing truth. (NRC 1989)*



Prompts

- How do you know when your students understand?
- What connections do you look for?
- What communication do you expect?

We understand something if we see how it is related or connected to other things we know."



Mathematical Learning

- Our view of mathematical learning influences how we think about teaching.

“Moving to classrooms that encourage understanding requires more than fiddling with one aspect, or adding more on to what is being done. It requires more substantive, long term changes. It also requires a change in attitude and beliefs as well as in practice and expectations.” (MS, p. xiv)



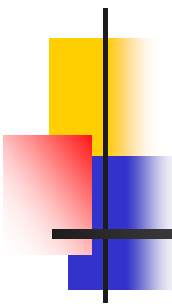
Making Sense

- What kinds of mathematical tasks do you present in your classroom?
- Do those task have the important characteristics of encouraging reflection and communication?
- Are there some ways that you can turn the task you are using now into ones that encourage understanding?
- What other aspects of mathematical tasks do you need to consider?
- Are there other mathematical tools needed in your classroom?
- How can you use mistakes as sites to encourage learning by everyone?
- How do you encourage students to share essential information?
- How do students determine the correctness of their mathematics?
- How do you involve each and every student in the sharing of their development of mathematical knowledge?



Student Engagement

- Students who lack understanding and must resort to memorizing are likely to feel little sense of satisfaction and are likely to withdraw from learning.
- Understanding breeds confidence and engagement; not understanding leads to disillusionment and disengagement.



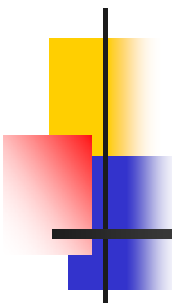
Is there a tradeoff between understanding and skill?

- Learning computational skills and developing conceptual understanding are frequently seen as competing objectives.
- However, if students are asked to work out their own procedures for calculating answers to arithmetic problems and to share their procedures with others, they will be necessarily engaged in reflecting and communicating.
- Sharing their work involves more than just demonstrating a procedure; it requires describing, explaining, justifying, and so on as they are asked questions by their peers.
- The primary goal of mathematics instruction is conceptual understanding.



Dimensions and Core Features of Classrooms

- A system of instruction which affords students opportunities to reflect and communicate is built on tasks that are genuine problems for students.
 - These are tasks which have no memorized rules, nor for which they perceive there is one right solution method.
- Appropriate tasks have 3 features
 1. They are problematic
 2. They most connect where the students are
 3. They must engage the students in thinking about important mathematical concepts
 1. That is, they must offer students the opportunity to reflect on important mathematical ideas, and to take something of mathematical value with them from the experience.

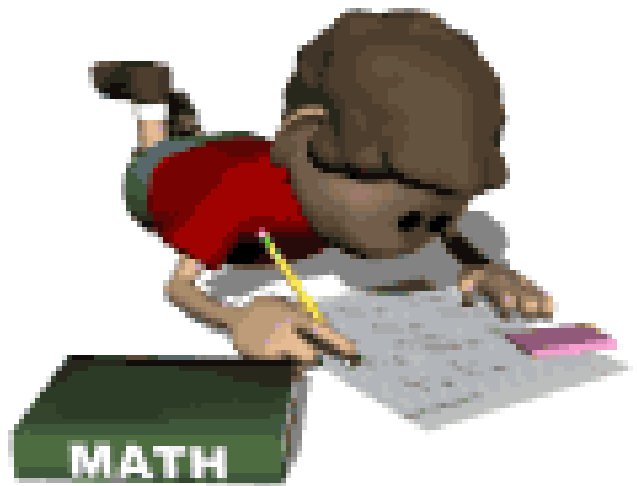


How students spend their time is determined by the tasks that they are asked to complete.

- If they spend most of their time practicing paper and pencil skills on sets of worksheet exercises, they are likely to become faster at executing these skills.
- If they spend most of their time watching the teacher demonstrate methods for solving special kinds of problems, they are likely to become better at imitating these methods on similar problems.
- If they spend most of their time reflecting on the way things work, on how various ideas and procedures are the same or different, on how what they already know relates to the situations they encounter, they are likely to build new relationships.
- That is, they are likely to construct understanding.
- The tasks make all the difference!

Time

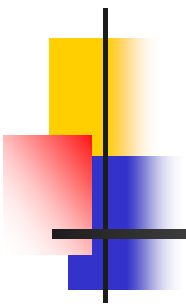
- If we want students to think that doing mathematics means solving problems, they will need to spend most of their time solving problems.



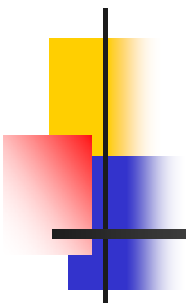
Learning

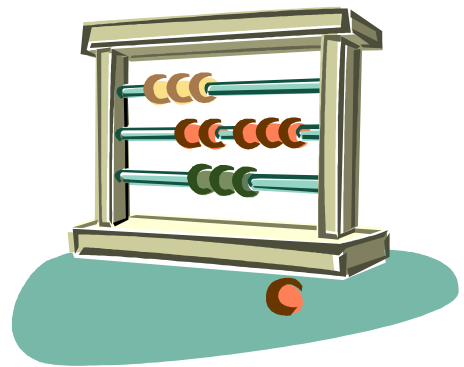
- The tasks must allow the students to treat the situations as problematic, something they need to think about, rather than a prescription they need to follow.



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- What is problematic about the tasks should be the mathematics rather than other aspects of the situation.



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- In order for students to work seriously on the task it must offer students the chance to use skills and knowledge they already possess.





The Role of the Teacher: Compare/ Contrast

Traditional	Constructivist
<p data-bbox="197 696 751 1096">Teacher feels responsible to tell students important mathematical information, demonstrate the procedures, and then to ask students to practice what they have seen and heard until they become proficient.</p> <p data-bbox="197 1115 758 1262">Understanding comes from listening carefully to what the teacher has to say.</p>	<p data-bbox="811 696 1379 1300">Is to select and pose appropriate sequences of problems as opportunities for learning, sharing information, what is essential for tackling problems, and facilitating the establishment of a classroom culture in which pupils work on novel problems individually and interactively and discuss and reflect on their answers and methods.</p> <p data-bbox="811 1319 1348 1566">The teacher relies on the reflective and conversational problem-solving activities of the students to drive their learning.</p> <p data-bbox="811 1586 1339 1881">Clear explanations and demonstrations from the teacher become less important than explanations and demonstration by the <u>students</u>.</p>

Reality: The real challenge is balance!

- How can we assist students in experiencing and acquiring mathematically powerful ideas but refrain from assisting so much that students abandon their own sense making skills in favor of following the teacher's directions?
- How can we handle the tension between supporting the initiative and problem-solving abilities of students and, at the same time, promoting the construction of mathematically important concepts and skills?





The Dilemma

- In most school settings the tendency of students to rely on their own problem-solving abilities is very fragile. If they sense that the teacher expects them to solve problems in a certain way, they will abandon their own efforts to understand and will search for ways to satisfy the teacher.
- Therefore, *how can we intervene in ways that push students' thinking forward and, at the same time, promote students' autonomy.*



Constructing Their Own Methods

- As students develop their own methods for solving problems, they develop general approaches for inventing specific procedures or adapting ones to fit new problems.
- Conversely, many students have trouble connecting the concepts they are learning with the procedures they are practicing. They often end up memorizing and practicing procedures they do not understand. This has damaging consequences, such as forgetting procedures, learning slightly flawed procedures without knowing it, or applying them rigidly without adjusting them for slightly different problems.

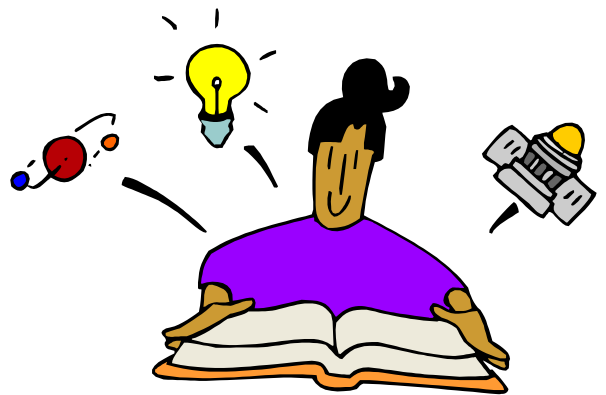


Application vs. Procedures

- If student separate their conceptual understanding from their procedures it means that they cannot solve problems very well.
- By trying to separate concepts and procedures directly, we artificially separate them.
- Without understanding, it is easy to forget procedures and distort them.

Begin with the end in mind!

- Therefore, an alternative is to begin with the problem!





The selection of appropriate tasks

- Includes thinking about how tasks are related, how they can be chained together to increase the opportunities for students to gradually construct their understanding
- Tasks are related if they allow students to see the same idea from different points of view, or if they allow students to build later solution methods on earlier ones. (MTB)



Careful Planning

- Teachers need to select sequences of tasks so that, over time, students' experiences add up to something important. Teachers need to consider the *residue* left by working on a *set* of tasks, not just individual tasks.
- This is hard!
 - Teachers need to have a good grasp of the important mathematical ideas and be familiar with students' thinking in order to select or make up appropriate individual tasks and coherent sequences or tasks. (MTB does this, in part, for us.)



Providing Relevant Information

- *How much information should teachers provide?*
- *How much should teachers tell students?*
- Dewey said that teachers should provide information if it is needed for students to continue their problem-solving efforts and they cannot readily find it for themselves, and if it is presented as something to consider and not a prescription to be followed.
- Information can and should be shared as long as it doesn't solve the problem, does not take away the need for students to reflect on the situation and develop solution methods that they understand.



Teachers can provide 2 kinds of information

1. Sharing math. conventions for recording and communicating actions and ideas
Like conventions in writing
2. Sharing alternative methods
This does not mean changing the students own methods to bring them more in line with the standard algorithm. (pg. 37)
3. Articulating ideas in students' methods, including restating what was said and clarifying ideas. (see Guiding Questions)



Adopt Appropriate Position of Authority

- There is no need for the teacher to have the final word of the correctness of the answer.
- The final word is provided by the logic of the subject and the students' explanations and justifications that are built on this logic.
- Inappropriate methods are usually challenged by other students



Teacher Make the System Work

- If teachers remove themselves from adjudicating the correctness of solutions, students will be more inclined to look to their own arguments to decide on correctness.
- They will be free to develop confidence in their own methods and their own monitoring skills for deciding whether something makes sense. They will be less inclined to try to uncover what the teacher wants or to guess what they answer key says.
- They will be free to focus their attention on developing justifications for their methods and solutions based on the logic of mathematics.



The Social Culture of the Classroom

- Means establishing certain expectations and norms for how students interact with each other about mathematics.
- *What features are needed to create a social culture that would support the kinds of tasks and reinforce the role of the teacher as prescribed?*
 1. *Ideas are currency in the classroom*
 2. *Students have autonomy as to methods used*
 3. *Mistakes are learning sites*
 4. *The authority for reasonability and correctness lies in the logic and structure of the subject*



Creating Cognitive Conflict

- “In mathematics class, cognitive conflict is created as students present their ideas and solution methods, defend them in the face of questions, and question others’ ideas. These experiences encourage students to deal with incongruities, to reevaluate their methods, to elaborate, to clarify, and reorganize their own thinking. Peer interaction is especially helpful because the differences in thinking are likely to be within a range that will generate genuine, fruitful conflict. Students often challenge each other in ways they can make sense of and deal with productively.” (p. 46)
- Vygotsky and Piaget



Norms of Healthy Social Culture

1. Discussions are about methods and ideas
2. Students choose their own methods and share them with others (defend, justify, HOTS)
All student need to listen and reflect (Luck of the Draw)
3. Mistakes are sites for learning
 1. "This is a serious issue, because if teachers view students' mistakes only in a negative way and try to prevent them, the system of instruction" described here, will collapse." As soon as we try to prevent students from making mistakes, we begin specifying the methods they should use. This removes the problematic nature of the task- the foundation of the system.
 2. A suggestion may be to reflect the problem back to the class for discussion, with the *explicit* aim of learning something new, seeing something that has bee hidden until now. Summarizing or affirming what was learned by discussing a particular error can emphasize how one can learn from mistakes.



Norms of Healthy Social Culture (continued)

4. Correctness is determined by logic of mathematics
 1. Students must learn to live with uncertainty as they evaluate the mathematical sense of a proposed method and solution.
 2. Holding correctness in suspension while the discussion and investigation continue runs counter to most practices in which teachers provide immediate feedback. But as Dewey strongly argued, experiencing this kind of uncertainty and even learning to enjoy it is an essential part of thoughtful problem solving. (p. 49)
 3. For students to accept the responsibility of deterring correctness and managing temporary uncertainties, they must develop confidence in themselves and their peers as capable sense-makers.
 4. **Every student needs the class and the class needs every student.**



Social Culture as Part of the System

- The way in which students approach tasks will depend as much on the culture of the classroom as on the tasks themselves.
- Tasks that are designed to be appropriately problematic for students may be changed by students or teachers, as they are implemented, into low-level conventional exercises. This, in turn may limit the kind of residues that students take with them from the activity.
- *How can we ensure this from happening in our classrooms?*
- *What are the hurdles?*
- *What does it mean when students are seemingly silent?*



Mathematical Tools as Learning Supports

- Language, materials, symbols
- *How do students make sense of tools?*
- *How do students develop meaning for tools?*
- In order for students to use tools wisely, they need to develop meaning for the tools.
- Tools can help keep records YMAW- Open number line, arrays for decomposition of numbers
- Vygotsky said that our thoughts are influenced by the tools we use.
- What is most important is not which tool a teacher chooses to introduce into the classroom, but rather that the teacher thinks carefully about the way in which students' thinking might be shaped by using particular tools. This kind of thinking is fruitful because it requires a thoughtful analysis of the mathematics in which students will be engaged and the kinds of understanding that might be left behind.



Equity and Accessibility

- Every student has the right to understand what they do in mathematics.
- Understanding supports improved performance for students at all levels.
- Equity, in part, means that each student is treated as an individual, and listening, *really listening*, is one of the best ways to encourage such treatment.

Make(ing) Sense!

- Learning with understanding is critical for all children.

